UCHBURCHAK BURCHAKLARIGA ICHKI CHIZILGAN AYLANALAR

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Annotatsiya: *Ushbu maqolada uchburchak burchaklariga ichki chizilgan aylana,ularga o’tkazilgan urunmalar o’tkazilishidan hosil bo’lgan burchaklar,qanday hosil qilingani va qanday qilib to’g’ri burchak,o’tkir burchak,o’tmas burchaklar hosil bo’lganini isbotlari keltirib o’tilgan.*

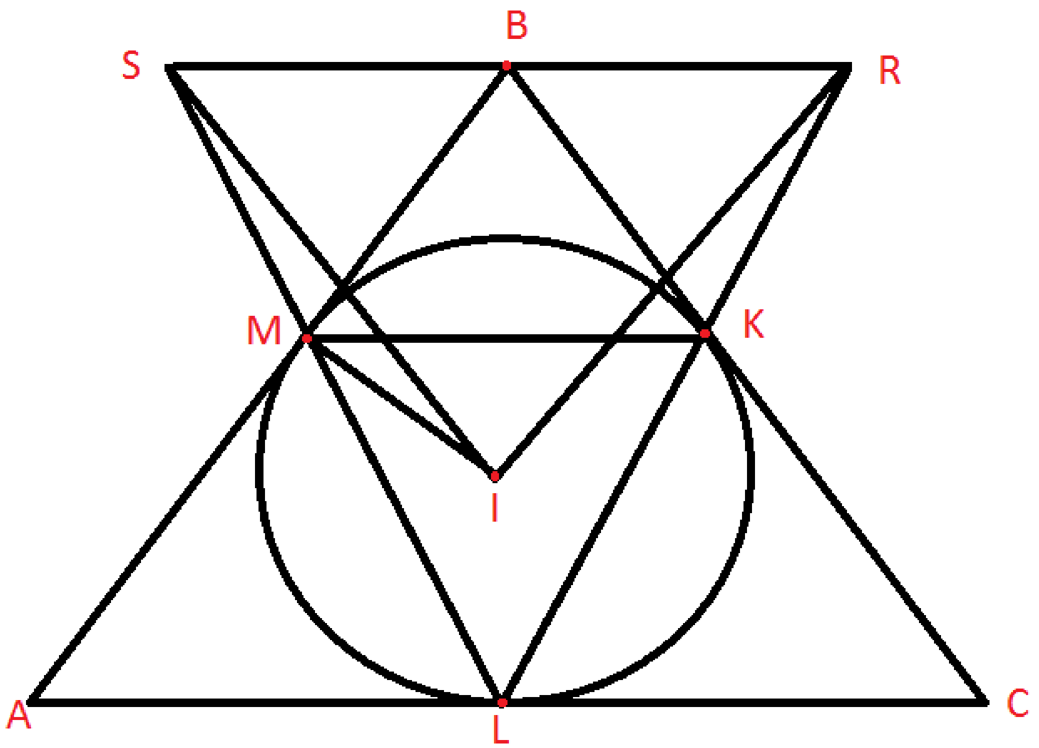
Kalit so’zlar: *To’g’ri burchakli uchburchak,uchburchak tomoniga o’tkazilgan urunma,vatar,uchburchakka ichki chizilgan aylana, uchburchakka tashqi chizilgan aylana.*

1. ABC uchburchakka ichki chizilgan aylana markazi I bo'lib, bu aylana BC, AC, AB tomonlarga mos ravishda K, L, M nuqtalarga urinadi. B uchdan MK ga parallel to‘g‘ri chiziq o'tkazilgan. Bu to‘g‘ri chiziq LM va LK ning davomlarini S va R nuqtalarda kesadi. Agar I-ichki chizilgan aylana markazi bo'lsa $\angle SIR < 90°$ ekanligini isbotlang.

Isboti: BM=BK, AM=AL, CL=SK. $\angle A = \alpha$, $\angle B = \beta$ $\angle C = \gamma$ bo`lsin.

$$\Rightarrow \angle BMK = \angle BKM = 90° - \frac{\beta}{2}, \quad \angle AML = \angle ALM = 90° - \frac{\alpha}{2}, \quad \angle CKL = \angle CLK = 90° - \frac{\gamma}{2},$$

$$\angle MBI = \frac{\beta}{2}, \quad BI \perp MK \Rightarrow (SR \,//\, MK)$$



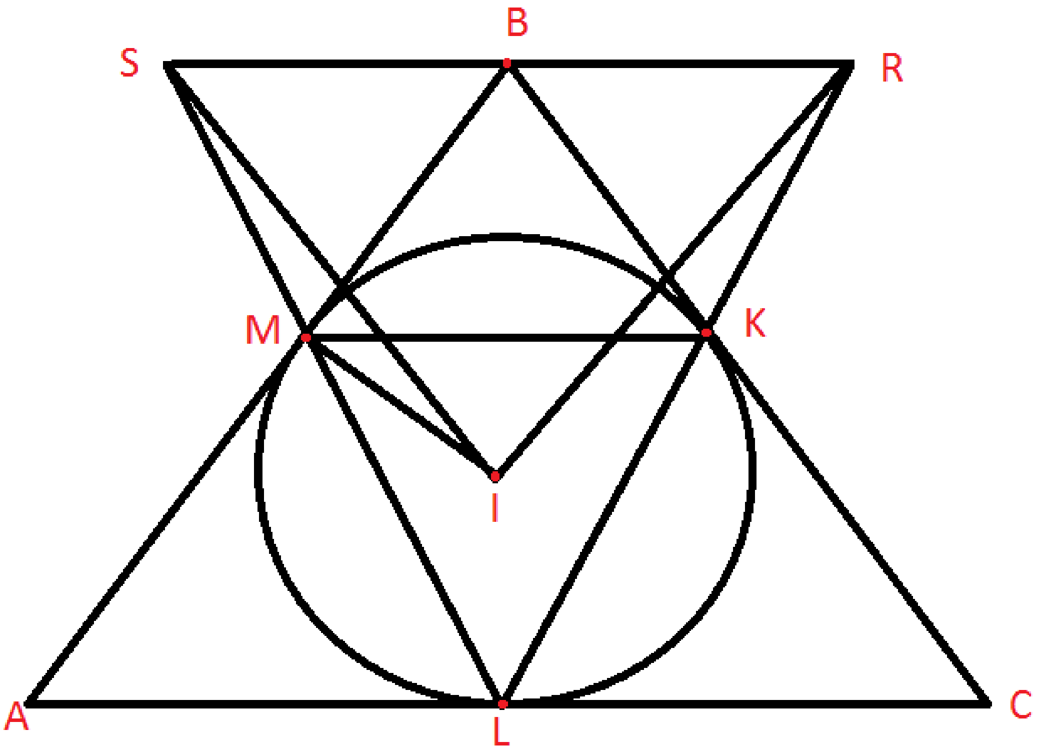
2. 0`tkir burchakli $\Delta ABC$ ga M nuqta BC ning o`rtasi. AM kesmada shunday P nuqta olinganki, bunda PM=MB. P nuqtadan $PH \perp BC$, $H \in BC$ qilib PH kesma o`tkazilgan. H nuqtadan BP va PC larga tushirilgan perpendikulyar AB va AC ni mos ravishda Q va R nuqtalarda kesadi. ΔQHR ga tashqi chizilgan aylana BC ga H nuqtada u rinishini isbotlang.

Isboti: $QR \cap PH = 0$ bo`lsin. QO=OR ni isbotlash yetarli, chunki $\angle QHR = 90°$, $\angle RHO = \varphi$ bo`lsin $\Rightarrow \frac{QO}{OR} = \frac{QH}{HR} \cdot \frac{\cos\varphi}{\sin\varphi} = \frac{QH}{HR} \cdot \frac{PC}{BP} \Rightarrow \frac{BP}{CP} = \frac{QH}{HR}$ ni isbotlash yetarli. $BP \cap AC = y$, $CP \cap AB = X \Rightarrow \frac{AY}{YC} \cdot \frac{MC}{MB} \cdot \frac{BX}{AX} = 1 \Rightarrow \frac{AX}{BX} = \frac{AY}{CY} \Rightarrow XYBC \Rightarrow \frac{BP}{PY} = \frac{BC}{XY} = \frac{PC}{PX} \Rightarrow$

$$\Rightarrow \frac{BP}{PC} = \frac{PY}{PX} = \frac{PB+PY}{PC+PX} = \frac{BY}{CX}$$

$\frac{HR}{BY} = \frac{HC}{BC}$, $\frac{CX}{QH} = \frac{BC}{BH} \Rightarrow \frac{QH}{HR} = \frac{BH}{CH} \cdot \frac{CX}{BY} = \frac{BH}{CH} \cdot \frac{PC}{PB} \Rightarrow \frac{BH}{HC} = \frac{PB^2}{PC^2}$ ni isbotlash kerak.

$1 + \frac{BH}{CH} = 1 + \frac{PB^2}{PC^2} \Leftrightarrow \frac{BC}{PC^2} = \frac{1}{HC} \Rightarrow PC^2 = HC \cdot BC$ ni isbotlash kerak. Bu esa to`g`ri burchakli ΔBPC da o`rinli. Isbotlandi.



3. ΔABC da $\angle A = 60°$. I-ichki chizilgan aylana markazi. $P \in BC$ bunda $BC = 3 \cdot PB$. $F \in AB$ bunda IF//AC U holda $\angle BEP = \angle$FBI ekanligini isbotlang.

Isboti: $\angle FBI = \angle CBI = \beta$, $\angle BFP = \alpha \Rightarrow \Delta FPB$ da $\frac{\sin\alpha}{BP} = \frac{\sin(2 \cdot \beta + \alpha)}{FB}$ (*)

ΔIFB da $\dfrac{\sin\angle BFI}{IB}=\dfrac{\sin\angle FIB}{FB}$ (**)

$$\left.\begin{array}{l}(*)\Rightarrow FB=BP\dfrac{\sin\left(2\cdot\beta+\alpha\right)}{\sin\alpha}\\ (**)\Rightarrow FB=IB\dfrac{\sin\left(60^\circ+\beta\right)}{\sin 60^\circ}\\ \dfrac{IB}{BP}=\dfrac{\sin\left(2\cdot\beta+\alpha\right)}{\sin\alpha}\cdot\dfrac{\sin\left(60^\circ+\beta\right)}{\sin 60^\circ}\end{array}\right\}\quad(1)$$

ΔBIC da $\dfrac{\sin\left(60^\circ-\beta\right)}{\sin 120^\circ}=\dfrac{IB}{BC}=\dfrac{IB}{3\cdot BP}\Rightarrow\dfrac{3\cdot\sin\left(60^\circ-\beta\right)}{\sin 120^\circ}=$

$$\frac{\sin\left(2\cdot\beta+\alpha\right)\cdot\sin 60^\circ}{\sin\alpha\cdot\sin\left(60^\circ+\beta\right)}\Rightarrow 4\cdot\sin\left(60^\circ-\beta\right)\cdot\sin\left(60^\circ+\beta\right)\cdot\sin\alpha=\sin\left(2\cdot\beta+\alpha\right)=$$
$$=2\cdot\sin\alpha\cdot\left(\cos 2\cdot\beta-\cos 120^\circ\right)=2\cdot\sin\alpha\cdot\cos 2\cdot\beta+\sin\alpha\Rightarrow\sin\left(2\cdot\beta-\alpha\right)=$$
$$=\sin\alpha\Rightarrow 2\cdot\beta=2\cdot\alpha\Rightarrow\beta=\alpha\Leftrightarrow\angle BFP=\angle FBI$$

Isbotlandi.

4. ΔABC da $\angle BAC=60^\circ$, $P\in|BC|, Q\in|AC|$. AP va BQ lar $\angle BAC$ va $\angle CBA$ larning bissektrissalari. Agar AQ+QB=PB+AB bo`lsa, ΔABC ning burchaklarini toping.

Yechish: AQ+QB=PB+AB $\Leftrightarrow\dfrac{AQ}{AB}+\dfrac{QB}{AB}=\dfrac{PB}{AB}+1$ (*)

ΔAQB da $\dfrac{AQ}{AB}=\dfrac{\sin\alpha}{\sin\left(60^\circ+\alpha\right)}$, $\dfrac{QB}{AB}=\dfrac{\sin 60^\circ}{\sin\left(60^\circ+\alpha\right)}$

ΔAPB da

$$\frac{PB}{AB}=\frac{\sin 30^\circ}{\sin\left(30^\circ+2\cdot\alpha\right)}\Rightarrow\frac{\sin 60^\circ+\sin\alpha}{\sin\left(60^\circ+\alpha\right)}=\frac{\sin 30^\circ+\sin\left(30^\circ+2\cdot\alpha\right)}{\sin\left(30^\circ+2\cdot\alpha\right)}\Rightarrow\frac{\cos\left(30^\circ-\frac{\alpha}{2}\right)}{\cos\left(30^\circ+\frac{\alpha}{2}\right)}=$$

$$=\frac{\sin 30^\circ+\sin\left(30^\circ+2\cdot\alpha\right)}{\sin\left(30^\circ+2\cdot\alpha\right)}\Rightarrow\sin\left(30^\circ+2\cdot\alpha\right)\cdot\cos\left(30^\circ-\frac{\alpha}{2}\right)=2\cdot\sin\left(30^\circ+\alpha\right)\cdot\cos\alpha\cdot\cos\left(30^\circ+\frac{\alpha}{2}\right)\Rightarrow$$

$$\Rightarrow\sin\left(30^\circ+2\cdot\alpha\right)\cdot\sin\frac{\alpha}{2}\cdot\cos\left(\frac{3\cdot\alpha}{2}+30^\circ\right)=\sin\left(30^\circ+\alpha\right)\cdot\cos\left(\frac{3\cdot\alpha}{2}+30^\circ\right)\Rightarrow$$

$$\Rightarrow\cos\left(\frac{3\cdot\alpha}{2}+30^\circ\right)\cdot\left(\frac{1}{2}-\cos\alpha\right)=0\Rightarrow\cos\alpha=\frac{1}{2}\,yoki\cos\left(\frac{3\cdot\alpha}{2}+30^\circ\right)=0$$

Agar $\cos\alpha=\dfrac{1}{2}$ bo`lsa $\alpha=60^\circ\Rightarrow\angle A+\angle B+\angle C=180^\circ+\angle C>180^\circ$

Ziddiyat. Demak $\cos\left(\dfrac{3\cdot\alpha}{2}+30^\circ\right)=0$ ekan.

$$\left(\frac{3\cdot\alpha}{2}+30^\circ\right)=90^\circ\Rightarrow 40^\circ\Rightarrow 2\cdot\alpha=80^\circ\Rightarrow 120^\circ-2\cdot\alpha=40^\circ$$

Javob: $\angle A=60^\circ$, $\angle B=80^\circ$, $\angle C=40^\circ$.

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